

不得使用計算機

請依照題號順序作答。所有數字必須化為最簡分數，未依規定者該題不予計分。除作圖外，答案限用黑色或藍色筆書寫。試題共三大題，第一大題為單選題，每題各佔5%。第二大題為多選題，每題各佔10%。第三大題為計算題，每題各佔15%。

一、單選題(占40分)

(說明: 第1題至第8題, 每題5個選項, 其中只有一個是最適當的答案, 請作答於答案卷首頁之選擇題作答區, 答案選項為大寫之(A)~(E)5個選項。

各題答對得5分; 未作答、答錯或畫記多於一個選項者, 該題以零分計算。)

1. 已知 $3x^2 + 2xy + y^2 = 1$, dy/dx 等於下列哪一個選項?

(A) $-\frac{3x+y}{y^2}$ (B) $-\frac{3x+y}{x+y}$ (C) $\frac{1-3x-y}{x+y}$ (D) $-\frac{3x}{1+y}$ (E) $-\frac{3x}{x+y}$ 。

2. 下述五個選項中, 那個選項中的 x 取值, 使得級數 $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ 收斂?

(A) 除了 $x=0$ 之外 (B) $x=3$ (C) $-3 \leq x \leq 3$ (D) $|x| > 3$

(E) 對所有的 x , 該級數都發散。

3. 已知 $df(x)/dx = g(x)$ 且 $h(x) = x^3$, 則 $\frac{d}{dx}f(h(x))$ 等於下列哪一個選項?

(A) $g(x^3)$ (B) $3x^2g(x)$ (C) $g'(x)$ (D) $3x^2g(x^3)$ (E) $x^3g(x^3)$ 。

4. 就下述之極限而言, 哪一個選項正確?

$$\lim_{h \rightarrow 0} \frac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$$

(A) 極限為 1 (B) 極限為 2 (C) 極限為 0 (D) 極限為 -1 (E) 極限不存在。

5. 若對所有的實數 x , $F'(x)$ 皆為連續函數, 則 $\lim_{h \rightarrow 0} \frac{1}{h} \int_3^{3+h^2} F'(x) dx$ 等於下列哪一個選項?

(A) 0 (B) $F(0)$ (C) $F(3)$ (D) $F'(0)$ (E) $F'(3)$ 。

6. Note that $z = f(x, y)$ where f is differentiable, $x = g(t)$, and $y = h(t)$. It is known that $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(2, 7) = 6$, and $f_y(2, 7) = -8$. Determine the value of dz/dt at $t = 3$.

(A) 62 (B) 30 (C) 32 (D) -2 (E) 1。

7. Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2) \exp(-t^2) dt$. Which of the following statements about g must be true?

I. g is increasing on the interval $(1, 2)$.

II. g is increasing on the interval $(2, 3)$.

III. $g(3) > 0$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III。

8. If $dy/dx = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$ (D) $\tan x + 5$ (E) $\tan x + 5e^x$ 。

二、多選題 (占 30 分)

(說明: 每題 10 分, 第 9 題至第 11 題, 每題有 5 個選項, 其中至少有一個是正確的選項, 請作答於答案卷首頁之選擇題作答區, 答案選項為大寫之 (A) ~ (E) 5 個選項。各題之選項獨立判定, 所有選項均答對者, 得 10 分; 每答錯 1 個選項, 扣 3 分; 所有選項均未作答或答錯多於 2 個選項者, 該題以零分計算。)

9. 當 $z = f(x, y)$, 且 $x = r \cos \theta$ 及 $y = r \sin \theta$ 。請問下列哪些選項是正確的?

(A) $\frac{\partial z}{\partial r} = \frac{\partial f}{\partial x} \cos \theta$

(B) $\frac{\partial z}{\partial \theta} = r(-\sin \theta \cdot \frac{\partial f}{\partial x} + \cos \theta \cdot \frac{\partial f}{\partial y})$

(C) $\frac{\partial^2 z}{\partial r^2} = \frac{\partial^2 f}{\partial x^2} \cos^2 \theta + \frac{\partial^2 f}{\partial x \partial y} \cos \theta \sin \theta$

(D) $\frac{\partial^2 z}{\partial r^2} = \frac{\partial^2 f}{\partial x^2} \cos^2 \theta + 2 \frac{\partial^2 f}{\partial x \partial y} \cos \theta \sin \theta + \frac{\partial^2 f}{\partial y^2} \sin^2 \theta$ (E) $\frac{\partial^2 z}{\partial r \partial \theta} = \frac{\partial^2 f}{\partial x \partial y} \cos \theta - \frac{\partial^2 f}{\partial x^2} \sin \theta$

10. 令 I 代表下列積分

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy dx.$$

請問下列哪些選項是正確的?

(A) $I = \int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^3 dr d\theta$

(B) $I = \int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^2 dr d\theta$

(C) $I = \iint_{\Omega} (x^2 + y^2) dA$, 其中 Ω 為以 $(1, 0)$ 為圓心, 半徑為 1 的圓盤

(D) $I = 4 \int_{-\pi/2}^{\pi/2} \cos^4 \theta d\theta$

(E) $I \leq 4\pi$

11. Let A be the region bounded above by the graph of $y = \sin(\pi x)$ and below by the graph of $y = x^3 - 4x$.

(A) $(2.1, 0.8)$ falls into the region A .

(B) The area of A can be represented by $\int_0^2 [\sin(\pi x) - (x^3 - 4x)] dx$.

(C) The area of A is equal to 4.

(D) The region A is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. The volume of this solid is equal to $\int_0^2 [\sin(\pi x) - (x^3 - 4x)]^2 dx$.

(E) The region A models the surface of a small pond. At all points in A at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. The volume of water in the pond is equal to $\int_0^2 (3 - x) [\sin(\pi x) - (x^3 - 4x)] dx$.

三、計算題 (佔 30 分)

(說明：本大題共有二題計算證明題，答案務必寫在答案卷上，並於題號欄標明題號(12, 13)與子題號((a)、(b))，同時必須寫出演算過程或理由，否則將予扣分。各小題配分標於題末。)

12. (15分) Let $f(x, y, z) = xyz$. Find the maximum and minimum values of f subject to the constraint $x^2 + y^2 + z^2 = 3$.

(a) Use Lagrange multiplier to obtain the system of equations to be solved. (7分)

(b) Find the maximum and minimum values of f . (8分)

13. (15分)

$$I(r) = \int \int_{\Omega_r} \exp [-(x^2 + 2xy + 3y^2)] dA$$

where $\Omega_r = \{(x, y) | x^2 + 2xy + 3y^2 \leq r^2\}$.

(a) Evaluate $I(1)$. (10分)

(b) Find $\lim_{r \rightarrow \infty} I(r)$. (5分)

試題隨卷繳回